Assignment 1 (ANSWER KEY)

1 MARK QUESTIONS

1 $A_R = \sqrt{[A_1^2 + A_2^2 + 2A_1 A_2\cos\phi]}$

$=\sqrt{[A^2 + (2A)^2 + 2A \times 2A\cos \pi/3]}$

$= \sqrt{7}\,A$

2 The resultant intensity at a point where phase difference is ϕ is

$I_R = I_1 + I_2 + 2[\sqrt{I_1 I_2}]\cos\phi$

$I_R = I + 4I + 2[\sqrt{I.4I}] \cos\phi = 5I+4I \cos\phi$

When $\phi=0$, $I_R = 5I+4I \cos0 = 9I$.

3 When $p = \lambda/4$, $\phi = \pi/2$

Therefore, $I = 4I_0 \cos^2 \pi/4 = 4I_0 \times ½ = 2I_0 = k/2$.

4 When $p = \lambda/6$, $\phi = \pi/3$ so $I=I_0 \cos^2 \pi/6 = 3I_0/4$.

5 Intensity at any point of an interference pattern is given by $I=2 I_0 (1+\cos \phi)$

Where I_0 is the intensity of either wave.

Here $\phi_P = 0$,

$\Phi_Q = 2\pi\ p/\lambda\ = 2\pi\ (\lambda/4)\ /\ \lambda\ \ =90$□

Therefore, $I_P /I_Q = [1 + \cos \phi_P] /[1 + \cos \phi_Q]$

$= [1 + \cos0] / [1 + \cos90$□$]$

$=[1 + 1] / [1 + 0] = 2/1 = 2:1$

6 Here $\lambda_1 = 630$ nm, $\beta_1 = 8.1$mm,

$\beta_2 = 7.2$mm, $\lambda_2 =?$

Fringe width, $\beta = D\lambda/d$

$\beta_2 / \beta_1 = \lambda_2 / \lambda_1$

Therefore, $\lambda_2 = [\beta_2 / \beta_1] \times \lambda_1$

$=7.2\text{mm} / 8.1\text{mm} \times 630\text{nm} = 560\text{nm}$.

7 Here, $\beta = 2.0$mm, $\mu = 1.33$

Refractive index of liquid,

μ = wavelength of light in vacuum/wavelength of light in liquid $= \lambda_v / \lambda_i$

$\lambda' = \lambda/\mu$

Fringe width in air,

$B = D\lambda/d$

Fringe width in liquid,

$\beta' = D\lambda'/d = D\lambda/d\mu = \beta/\mu = 2.0mm/1.33 = 1.5mm$

8 $X = n\lambda_1 D/d = (n + 1)\lambda_2 D/d$

$n\lambda_1 = (n + 1)\lambda_2$

$n = 3$

therefore, $x = 3D\lambda_1/d = 12mm$

9 In first case, $X = D_1\lambda_1/d$

In second case, $X = D_2\lambda_2/ d/2$

Therefore, $D_1\lambda_1/d = D_2\lambda_2/ d/2$

$D_1/D_2 = 2\cdot\lambda_2/\lambda_1 = 2 \times 600/400 = 3/1 = 3:1$

10 position of 1^{st} minimum $x_1 = D\lambda / d$

Slit width d= 0.24 mm

11 Angular width of central maximum

$2\theta = 2\lambda / d$

12 $d \sin\theta = \lambda$

$D = 1.3 \times 10^{-6}$ m

13 Frequency is the characteristics of the source while wavelength is the characteristics of medium when monochromatic light travels from one medium to another its speed changes so it's wavelength changes but frequency remain same.

14 1. The two rays must be continuous.

2. They should have a constant or zero phase difference.

15 $B = D\lambda / d$

As the separation D between the two slits decreases French width increases

16 When the distance between the slit and the screen is double the angular separation remains unchanged

17 spherical wave front

18 Path difference$\Delta = [2n + 1]\lambda/2$

Where n = 0,1,2.....

19 Both the fringe width and angular separation decreases.

MCQ ANSWERS

20. A
21. C
22. D
23. C
24. B
25. D
26. D
27. D
28. A
29. D

ANSWERS OF 2 MARKS QUESTIONS

30. Fringe width = $D\lambda / d = Dc/d\nu$

i) When D decreases fringe width decreases.

ii) When the frequency ν is increased fringe width increases.

31. (a)(i) Distance of nth bright fringe from central maximum = $nD\lambda / d$ = 6mm

(ii) Distance of nth dark fringe from central maximum = $[2n-1]D\lambda /2d$ = 4.5 mm

(b) when D increases fringe width increases

32. (i)Light is emitted by the individual items and not by the bulk of matters acting as a whole.

(ii) Even the tiniest source consists of millions of atoms and the emission of light by them takes place independently.

(iii) Even an atom emits and unbroken wave of about 10^{-8} second due to its transition from higher energy state to lower energy state.

The millions of atoms of a source cannot emit wave in the same phase. The face difference and hence the interference pattern changes 10 times in 1 second. Our eyes cannot see such rapid changes and a uniform illumination is seen on the screen. So independent light sources cannot produce a sustained interference.

33. In an interference pattern the average intensity is at the point of maxima and minimum are such that

I(average) α $a_1^2 + a_2^2$

If there is no interference between the light waves from the two sources then the intensity at every point would be same that is

$I = I_1 + I_2$ α $a_1^2 + a_2^2$

Which is same as I (average) in the interference pattern. So, there is no violation of law of conservation of energy in the interference.

34. Fringe width is inversely proportional to distance between two slits.

(i) when the two coherent sources are placed in finitely close to each other the fringe width becomes very large. Even a single fringe may occupy the entire screen. The interference pattern is not observable.

(ii) as the distance between the sources is increased the French with goes on decreasing. At very large separation it becomes too small to be detected. The interference pattern cannot be observed.

35. Fringe width $\beta = D\lambda / d = Dc/d\nu$

(i) wave length of light in water decreases so fringe width decreases.

(ii)When light of small frequency is used, fringe width increases.

36. Fringe width $\beta = D\lambda / d$

$\lambda_1 / \lambda_2 = \beta_1 / \beta_2$

$\lambda_2 = \lambda_1\beta_2 / \beta_1 = 7.2 \times 630/8.1 = 560$nm

37. $\frac{\beta 2}{\beta 1} = \frac{D\lambda 2/2d}{D\lambda 1/d} = \lambda_2 / 2\lambda_1$

$\beta_2 = [7500 \times 0.8] / [2 \times 6000] = 0.5$ mm

38. Ratio $r = a_1 / a_2 = \sqrt{[I_1 / I_2]} = \sqrt{[4/9]} = 2/3$

$\frac{I_{max}}{I_{min}} = [\frac{r+1}{r-1}]^2 = 25:1$

39. $\Lambda = 6000$ Å , $a = 1 \times 10^{-4}$m , D = 1.5m

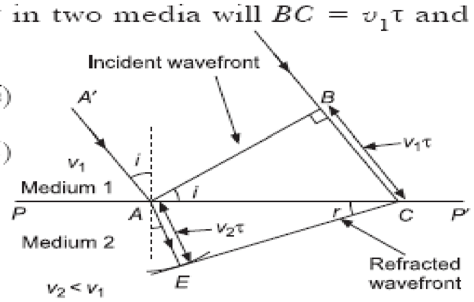
The distance between two dark lines on either side of central maximum = width of central maximum= $2\lambda D/a = (2 \times 6000 \times 10^{-10} \times 1.5) / (1 \times 10^{-4})$

$= 1.8 \times 10^{-2}$m= 1.8cm

3 MARKS QUESTIONS ANSWERS

40.

If v_1 and v_2 are the speeds of light in media 1 and 2 respectively, then distance travelled by light in a small time internal τ in two media will $BC = v_1\tau$ and $AE = v_2\tau$ respectively.



In ΔABC, $\sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC}$ ...(i)

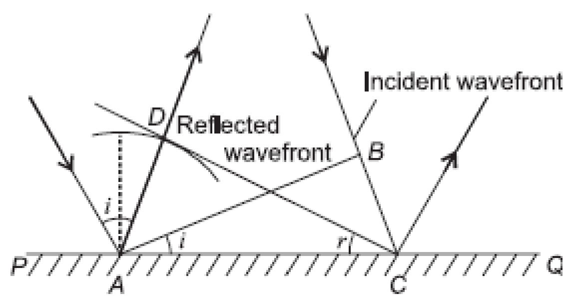
In ΔAEC, $\sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC}$ ...(ii)

Combining equations (i) and (ii), we get

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \mu_{21}$$

The above relation is known as Snell's law.

41. A wavefront is the locus of all points oscillating in same phase. A figure showing reflection of a plane wavefront using Huygen's construction is given below. In the figure AB is incident wavefront and *CD* is reflected wavefront. If *v* is speed of the wave in the medium and t is the time taken by the wavefront to cover distance *BC*, then



$$BC = vt$$

Obviously, $AD = vt$

As ΔABC and ΔADC are congruent.

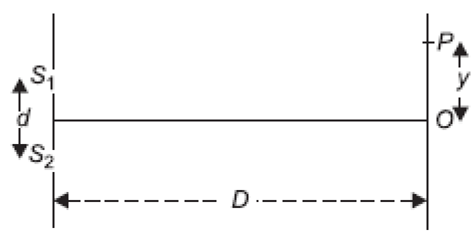
$\therefore$ $\angle i = \angle r$

42

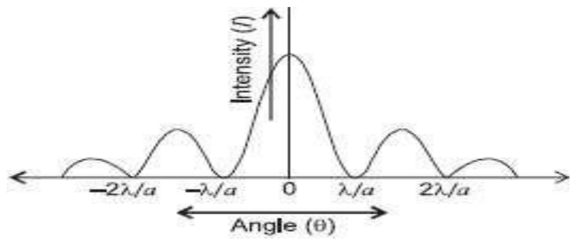
Given: $y = \frac{\lambda D}{3d}$

As $\Delta P = \frac{yd}{D} \Rightarrow \Delta P = \frac{\lambda}{3}$ or $\Delta\phi = \frac{2\pi}{3}$

$\therefore$ $I = I_0 \cos^2 \Delta\phi = I_0 \left(\cos\frac{2\pi}{3}\right)^2 = \frac{I_0}{4}$



44. When a plane wavefront is incident on a single slit, all the point sources of light constituting the wavefront are in same phase. The wavelets coming out from the wavefront might meet over the screen with some path difference, i.e., a phase difference is introduced between them.The brightness at a point on the screen depends on the phase difference between the wavelets meeting at the point. We imagine that the slit is divided into smaller parts and the wavelets coming out from these portions meet and superpose on the screen with proper phase difference.



The wavelets from different parts of the wavefront, incident on the slit, meet with zero phase difference to constitute a central maximum. In case of secondary maxima, there are some wavelets meeting the screen out of phase, thus, reducing intensity of secondary maxima.

45.

The resultant intensity is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi, \text{ where } I_1 = I,\ I_2 = I + \delta I$$

At maxima, $\cos\phi = 1$

$$\therefore \quad I_{max} = I + I + \delta I + 2\sqrt{I(I+\delta I)}$$

$$\therefore \quad I_{max} = 2I + 2I = 4I \qquad (\because\ \delta I << I)$$

At minima, $\cos\phi = -1$

$$I_{min} = I + I + \delta I - 2\sqrt{I(I+\delta I)}$$

$$I_{min} = 2I + \delta I - 2\left[I^2\left(1+\frac{\delta I}{I}\right)\right]^{1/2} = 2I + \delta I - 2I\left[1 + \frac{1}{2}\frac{\delta I}{I} - \frac{1}{8}\left(\frac{\delta I}{I}\right)^2 + \ldots\right]$$

Neglecting the higher power, we get $I_{min} = \frac{2I}{8}\left(\frac{\delta I}{I}\right)^2 = \frac{1}{4}\frac{(\delta I)^2}{I}$

46. Linear width of central maximum β= 2Dλ/d

Angular width of central maximum β/D = 2λ/d

(i) When slit width d decreases, angular width increase.

(ii) When distance D between the slit and screen is increased, angular width does not change.

(iii) When light of smaller wavelength is used, angular width decreases.

47. λ=550 nm, d= 0.1 mm,D = 1.1m, ω = ?, β = ?

using ω = 2θ= 2λ/d, we get ω= .011 rad

using $\beta = 2\lambda D/d$,we get β = 12.1 mm

When the screen is moved to 2.2 m from the slit, the angular width will not change, linear width will increase.

48. Diffraction of light is the phenomenon of bending of light round the corners of an opaque obstacle and spreading into the regions of the geometrical shadow.

Diffraction	Interference
The phenomenon of interaction of light coming from different parts of the same wave front is called diffraction.	The phenomenon of non-uniform distribution of light energy (wave) due to the superposition of coherent sources of light is called interference.
In diffraction, the widths of fringes are not equal.	In interference, the width of fringes are equal.
Bands are very less in number.	Bands are very large in number.
Dark fringes in diffraction are not completely dark.	Dark fringes in interference are perfectly dark.

49. λ = 6000 Å , $a = 1 \times 10^{-4}$m , D = 1.5m

The distance between two dark lines on either side of central maximum = width of central maximum= $2\lambda D/a = (2 \times 6000 \times 10^{-10} \times 1.5) / (1 \times 10^{-4})$

$= 1.8 \times 10^{-2}$m= 1.8cm

i.e. $\sin\theta = \dfrac{\Delta P}{a}$

From $\Delta QOO'$, $\tan\theta = \dfrac{Y}{D}$

For a small angle, $\sin\theta \approx \tan\theta$ i.e. $\Delta P = \dfrac{Ya}{D}$

For bright fringes, $\Delta P = n\lambda$

Thus, $\dfrac{Y_n a}{D} = n\lambda \Rightarrow Y_n = \dfrac{n\lambda D}{a}$

For dark fringes, $\Delta P = (2n-1)\dfrac{\lambda}{2}$

Thus, $\dfrac{Y'_n a}{D} = (2n-1)\dfrac{\lambda}{2} \Rightarrow Y'_n = \dfrac{(2n-1)D\lambda}{2a}$

The separation between two consecutive dark or bright fringes is called fringe width.

i.e., $\beta = Y_n - Y_{n-1} = \dfrac{n\lambda D}{a} - \dfrac{(n-1)\lambda D}{a} = \dfrac{\lambda D}{a}$

Here β is the fringe width.

(b) Given: For an interference pattern, $\dfrac{I_{min}}{I_{max}} = \dfrac{9}{25}$, $\dfrac{I_1}{I_2} = ?$

Here $\dfrac{I_1}{I_2} = \dfrac{a_1^2}{a_2^2}$ and $\dfrac{I_{min}}{I_{max}} = \dfrac{\left(\dfrac{a_1}{a_2}-1\right)^2}{\left(\dfrac{a_1}{a_2}+1\right)^2}$

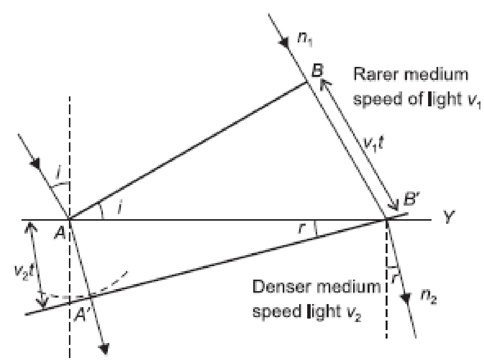
$$\therefore \quad \frac{\left(\frac{a_1}{a_2}-1\right)^2}{\left(\frac{a_1}{a_2}+1\right)^2} = \frac{9}{25} \Rightarrow \frac{\frac{a_1}{a_2}-1}{\frac{a_1}{a_2}+1} = \frac{3}{5}$$

$$\Rightarrow \quad 3\left(\frac{a_1}{a_2}+1\right) = 5\left(\frac{a_1}{a_2}-1\right) \Rightarrow \frac{a_1}{a_2} = 4$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{16}{1}$$

5 MARKS QUESTIONS

50. (a) According to the Huygens's principle, each point of the wavefront is the source of secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. A common tangent to all the wavelets in the forward direction gives the new position of



wavefront at a later time.

From $\Delta ABB'$, $\sin i = \dfrac{BB'}{AB'} = \dfrac{v_1 \times t}{AB'}$ ...(i)

From $\Delta AA'B'$, $\sin r = \dfrac{AA'}{AB'} = \dfrac{v_2 \times t}{AB'}$ ...(ii)

$\therefore$ $\dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2}$

We know $n_1 = \dfrac{c}{v_1}$ and $n_2 = \dfrac{c}{v_2}$

where n_1 and n_2 are the refractive indices of the 1^{st} and 2^{nd} media.

So, $n_1 \sin i = n_2 \sin r$

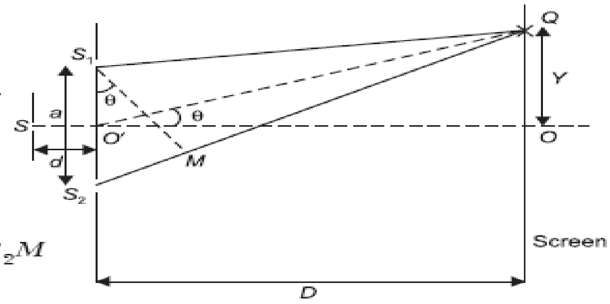
which is Snell's law of refraction.

(b) (i) Frequency remains the same. When the light of particular frequency is incident it interacts with the atoms of the matter, which further causes forced oscillations. As the frequency of charged oscillator and the frequency of wave emitted by charged oscillator is same, therefore the frequency of reflected and refracted light is same.
(ii) No, energy carried by a light wave does not depend on its speed. Instead it depends on its amplitude.

51. (a) (i) 'Two independent monochromatic sources of light cannot produce a sustained interference pattern'. Give reason.
(ii) Light waves each of amplitude a and frequency *n*, emanating from two coherent light sources superpose at a point. If the displacements due to these waves is given by $y_1 = a \cos t$ and $y_2 = a \cos (\omega t + \phi)$, what is the phase difference between the two, obtain the expression for the resultant intensity at the point.
(b) In Young's double slit experiment, using monochromatic light of wavelength, the intensity of light at a point on the screen where path difference is, is *K* units. Find out the intensity of light at a point where path difference is $\dfrac{\lambda}{3}$.

52.

(*a*) When waves from the slits meet at a point on the screen with same phase, the maxima are obtained and with a phase difference of π, the minima are obtained. According to the Young's experiment, the path difference between the waves is given by



$$\Delta P = S_2Q - S_1Q = S_2M$$

i.e. $\sin\theta = \frac{\Delta P}{a}$

From $\Delta QOO'$, $\tan\theta = \frac{Y}{D}$

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Thus, $\frac{Y'_n a}{D} = (2n - 1)\frac{\lambda}{2} \Rightarrow Y'_n = \frac{(2n-1)D\lambda}{2a}$

The separation between two consecutive dark or bright fringes is called fringe width.

i.e., $\beta = Y_n - Y_{n-1} = \frac{n\lambda D}{a} - \frac{(n-1)\lambda D}{a} = \frac{\lambda D}{a}$

Here β is the fringe width.

(*b*) Given: For an interference pattern, $\frac{I_{min}}{I_{max}} = \frac{9}{25}$, $\frac{I_1}{I_2} = ?$

Here $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$ and $\frac{I_{min}}{I_{max}} = \frac{\left(\frac{a_1}{a_2} - 1\right)^2}{\left(\frac{a_1}{a_2} + 1\right)^2}$

$$\therefore \quad \frac{\left(\frac{a_1}{a_2} - 1\right)^2}{\left(\frac{a_1}{a_2} + 1\right)^2} = \frac{9}{25} \Rightarrow \frac{\frac{a_1}{a_2} - 1}{\frac{a_1}{a_2} + 1} = \frac{3}{5}$$

$$\Rightarrow \quad 3\left(\frac{a_1}{a_2} + 1\right) = 5\left(\frac{a_1}{a_2} - 1\right) \Rightarrow \frac{a_1}{a_2} = 4$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{16}{1}$$